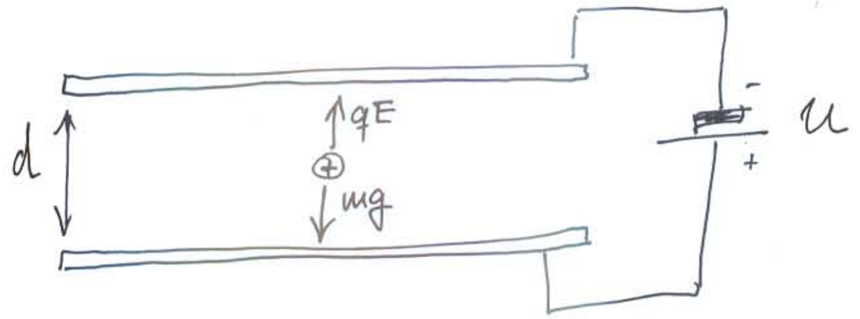
FIS 1532 Pauta I1

Problema 1 :



Fuerza Coulomb compensa fuerza grav.:

$$qE = mg$$

$$\hookrightarrow \boxed{E = \frac{m}{q} g} \qquad (0{,}5\,pt)$$

Voltaje entre placas planas:

$$\boxed{U = dE = d\frac{m}{q} g} \qquad (0{,}5\,pt)$$

Valor numérico:

$$U = 0{,}01\,m \cdot \frac{0{,}001\,kg}{4{,}9 \cdot 10^{-3}\,C} \cdot 9{,}8\,\frac{m}{s^2}$$

$$= 0{,}01 \cdot \frac{9{,}8}{4{,}9}\,V = \underbrace{0{,}02}_{(0{,}5pt)}\,V = 20\,mV$$

unidad (0,5pt)

2) Capacidad por definición:

$$\boxed{C = \frac{Q}{U}}$$ (1pt)

Valor numérico:

$$C = \frac{1{,}0 \cdot 10^{-12}\,C}{2 \cdot 10^{-2}\,V} = 0{,}5 \cdot 10^{-10}\,F$$

$$= \underbrace{50 \cdot 10^{-12}}_{(0{,}5pt)}\,F = 50\,pF$$

unidad (0.5pt)

3) Capacidad de dos placas planas:

$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$ (0,5pt)

$$\hookrightarrow \boxed{A = \frac{dC}{\varepsilon_0}}$$ (0,5pt)

Valor numérico:

$$A \cong \frac{0{,}01\,m \cdot 5 \cdot 10^{-11}\,F}{9 \cdot 10^{-12}\,F/m}$$

$$= \frac{50}{9} \cdot 10^{-14+12} \ m^2$$

$$50 : 9 = 5{,}55\ldots \cong 5{,}6$$

$$\begin{array}{l} 50 \\ \underline{45} \\ \ \ 50 \\ \ \ \ \vdots \end{array}$$

$$A \cong \underbrace{5{,}6 \cdot 10^{-2}}_{(0{,}5\,pt)} \ m^2$$

↑ unidad (0,5 pt)

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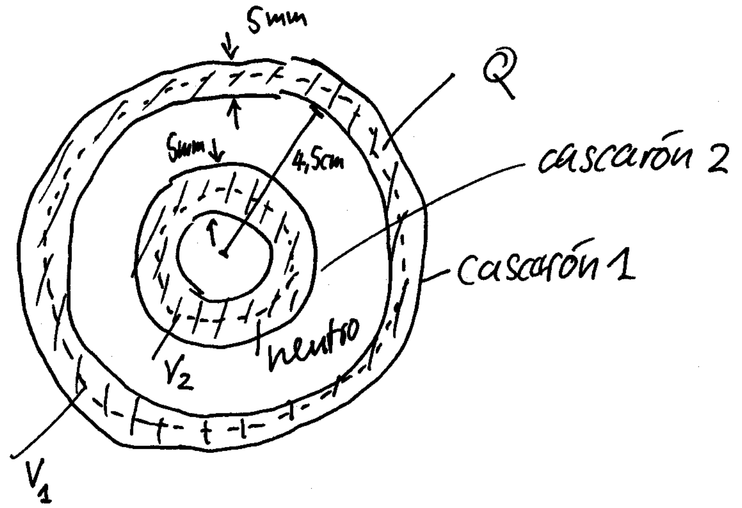
Problema 2:



1) V_1 : volumen esférico con superficie en casc. 1

ley de Gauss:

$$\oint_{\partial V_1} d\vec{A}\cdot \underbrace{\vec{E}(\vec{r})}_{=0\ (\text{metal})} = \frac{1}{\varepsilon_0}\int_{V_1} dV\, \rho(\vec{r}) = 0$$

(1pt)

$$\int_{V_1} dV\, \rho(\vec{r}) = 0 = Q_{1,int} + \underbrace{(Q_{2,ext} + Q_{2,int})}_{=0\ (\text{casc.2 neutro})}$$

$$\hookrightarrow \boxed{Q_{1,int} = 0}$$

$$\Downarrow$$

$$\boxed{\sigma_{1,int} = 0}$$ (1pt)

densidad $\boxed{\nu_{1,ext} = \dfrac{Q}{4\pi\, r_{1,ext}^2}}$ (0,5 pt)

$r_{1,ext} = 5\ \text{cm}$

$\Rightarrow$ Valor numérico:

$$\nu_{1,ext} = \frac{31{,}4 \cdot 10^{-6}\ \text{C}}{3{,}14 \cdot 4 \cdot (0{,}05)^2\ \text{m}^2} = \frac{10 \cdot 10^{-6}\ \text{C}}{4 \cdot 0{,}0025\ \text{m}^2}$$

$$= \frac{10^{-5}\ \text{C}}{0{,}01\ \text{m}^2} = 10^{-3}\ \frac{\text{C}}{\text{m}^2}$$

con unidad (0,5 pt)

$$= 1\ \frac{\text{mC}}{\text{m}^2}$$

2) V_2: vol. esf. con superficie en casc. 2

ley de Gauss

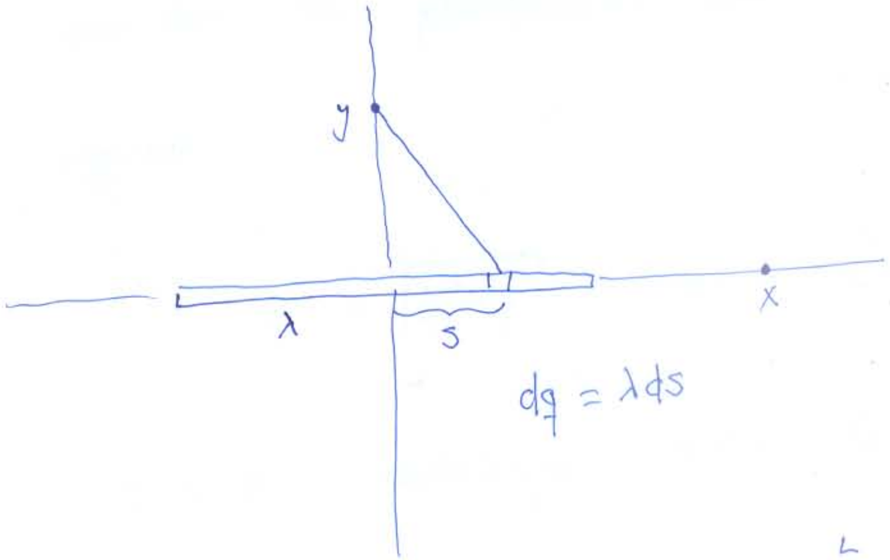
$$\oint_{\partial V_2} d\vec{A} \cdot \underbrace{\vec{E}(\vec{r})}_{=0\ (\text{metal})} = \frac{1}{\varepsilon_0} \int_{V_2} dV\, \rho(\vec{r}) = 0$$

(1 pt)

$$\int_{V_2} dV\, \rho(\vec{r}) = 0 = Q_{2,int} \rightarrow \boxed{\sigma_{2,int} = 0} \quad \text{(1pt)}$$

Casc. 2 es neutro $\Rightarrow Q_{2,ext} = 0 \Rightarrow \boxed{\sigma_{2,ext} = 0}$ (1pt)

$$\rightarrow \sigma_{2,int} = \sigma_{2,ext} = 0$$



$dq = \lambda ds$

$d\varphi = \frac{k\,dq}{r}$

$$d\varphi(y,s) = \frac{k\lambda ds}{\sqrt{y^2+s^2}} \qquad \varphi(y) = \int_{-L}^{L} \frac{\lambda ds}{\sqrt{y^2+s^2}}$$

(a) $$\varphi(y) = k\lambda \ln\left(\frac{L+\sqrt{L^2+y^2}}{-L+\sqrt{L^2+y^2}}\right)$$

Ahora calculemos $\varphi(x)$

(b) $$d\varphi(x) = \frac{k\lambda\, ds}{x-s} \qquad \varphi(x) = \int_{-L}^{L} \frac{k\lambda\, ds}{x-s}$$

$$\varphi(x) = -k\lambda \ln(x-s)\Big|_{-L}^{L}$$

$$= -k\lambda \ln\frac{x-L}{x+L}$$

$$= k\lambda \ln\frac{x+L}{x-L}$$

Para que los dos potenciales sean iguales

se requiere:

$$X = \sqrt{L^2 + y^2} \quad \rightarrow \quad x^2 - y^2 = L^2$$

Si $L = 0$ entonces $x = y$ lo cual es

razonable.